

1. What patterns do you notice in the figure?

There are three main patterns:

1. The number of columns in the middle of each figure corresponds to one less than the diagram number.
2. The number of tiles in each column in the middle of each figure corresponds to one more than the diagram number.
3. The most obvious pattern is that each figure, regardless of diagram number, has the 2 tiles on the sides of the columns.

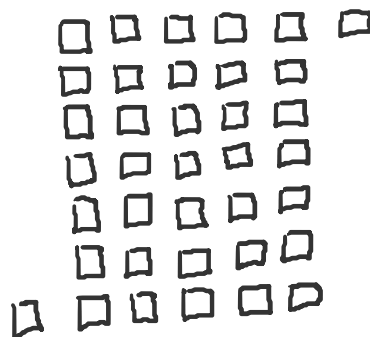
2. Sketch the next figure in the sequence.

Diagram #6

5 Columns

7 Tiles in

Each Column



3. Determine an equation for the total number of tiles in any figure in the sequence. Explain your equation and show how it relates to the visual diagram of the figure. Try to find as many different equations as possible!

To create an equation, it may help to create a table first.

Diagram	Columns	Tiles in Each Column	Total Tiles in Columns	Tile Constant	Total Tiles
1	0	—	—	2	2
2	1	3	3	2	5
3	2	4	8	2	10
4	3	5	15	2	17
5	4	6	24	2	26
6	5	7	35	2	37

Now to look for a pattern to write as an equation may be difficult looking at only the total number of tiles. But, once you realize that every diagram starts with 2 tiles, you can look at the total tiles in each column.

The connection to get this number is to multiply the column number times the tiles in each column.

Looking further, the connection between the diagram and the column is one less:

$$\begin{aligned}\text{Column Number} &= \text{Diagram Number} - 1 \\ &= \boxed{n - 1}\end{aligned}$$

The connection between the diagram and the tiles in each column is one more:

$$\begin{aligned}\text{Tiles in each Column} &= \text{Diagram Number} + 1 \\ &= \boxed{n + 1}\end{aligned}$$

→ For any diagram number, to find the total tiles in all the columns, we multiply the two above  $(n-1)(n+1)$ . Then, to find the total tiles overall, we take this number and add 2.

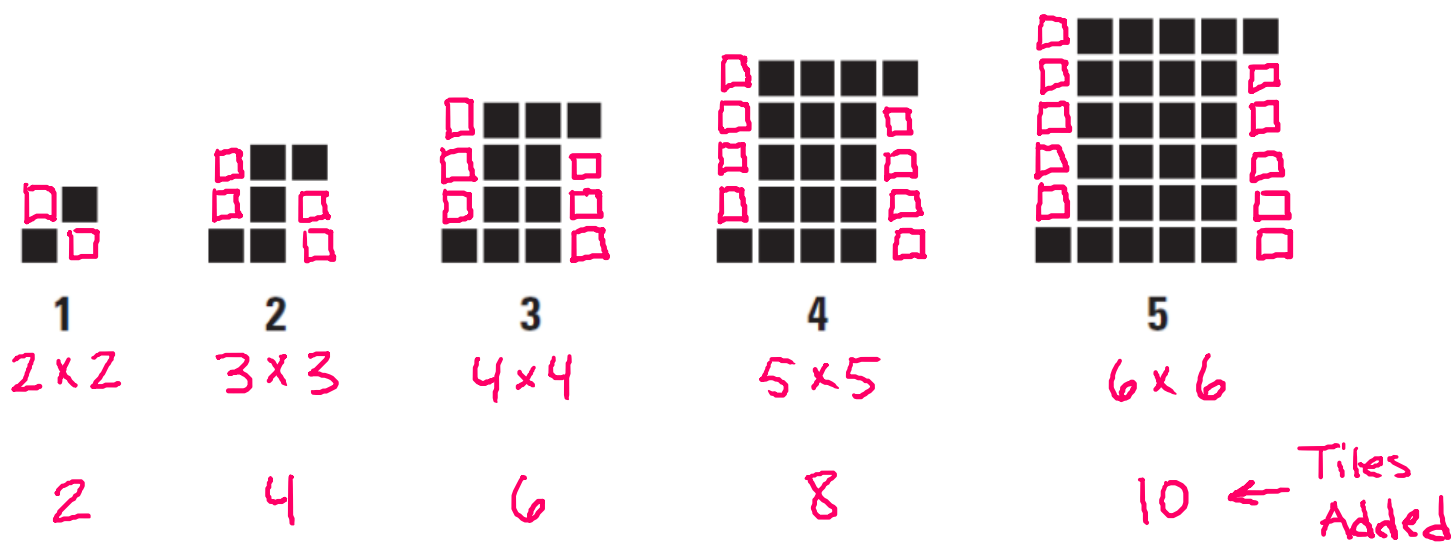
Total Number of  
Tiles for any  
Diagram Number

$$\begin{aligned}&= \overbrace{(n-1)(n+1)} + 2 \\ &= n^2 + \cancel{1n} - \cancel{1n} - 1 + 2 \\ &\rightarrow = \boxed{n^2 + 1}\end{aligned}$$

Some of you might have been able to see this pattern and determine this equation by looking at the total tiles in our table above.

4. Sarah wrote the expression  $(n + 1)^2 - 2n$ . Can you explain how her expression relates to the diagram?

In order to square something in math, the same number times itself is needed. Sarah took the diagram and created squares, and then subtracted those tiles she added out:



The dimensions for the square of each diagram is one more than the diagram number, which is where the  $(n+1)$  comes from.

To find the total tiles in each diagram, square  $(n+1)$  since you are multiplying the number times itself...  $(n+1)^2$ .

Next, Sarah needed to subtract out the tiles she added and the connection between the diagram number and the number of tiles added is doubled...  $2n$ .

Which is how Sarah got  $(n+1)^2 - 2n$ .

5. If you knew the figure had 9802 tiles in it, how could you determine the figure number? Explain.

The equation I came up with to determine the total amount of tiles for any diagram number was:

$$\begin{array}{l} \text{Total \#} \\ \text{of Tiles} \end{array} = n^2 + 1 \quad \left( n \text{ represents diagram number} \right)$$

$$\rightarrow 9802 = n^2 + 1 \quad \text{Solve for } n$$

$$\begin{array}{r} -1 \\ \hline 9801 \end{array} = \sqrt{n^2}$$

$$\rightarrow n = 99$$

The 99<sup>th</sup> diagram has 9802 tiles.